Bargaining Theory and Solutions

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Networks: Technology, Economics, and Social Interactions

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Outline

- Bargaining Problem
- Bargaining Theory
  - Axiomatic Approach
  - Strategic Approach
- Nash Bargaining Solution (Axiomatic)
- Rubinstein Bargaining Solution (Strategic)
- Conclusion

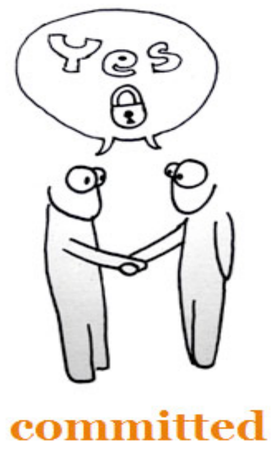
Bargaining Problem

- Bargaining is one of the most common activities in daily life.
- Examples:
  - Price bargaining in an open market;
  - Wage bargaining in a labor market;
  - Score bargaining after an examination;
  - ... ...

bargaining

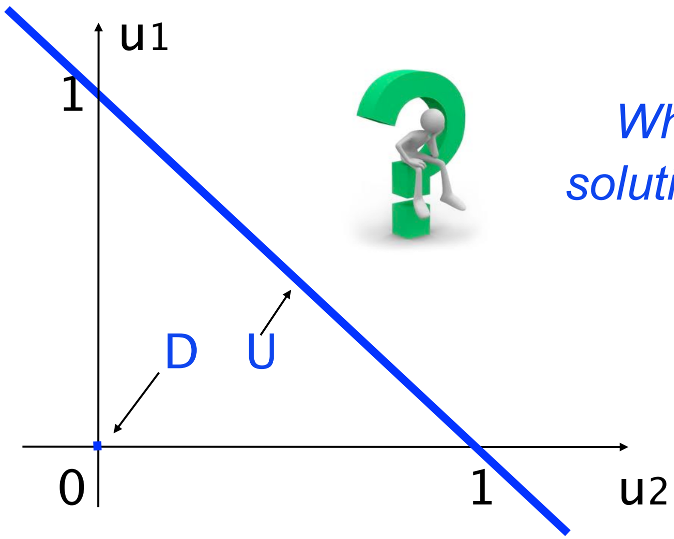
Bargaining Problem

- Bargaining problems represent situations in which:
  - There is a common interest among players to address a mutually agreed outcome (agreement).
  - Players have specific objectives (utility or payoff).
  - No agreement may be imposed on any player without his approval, i.e., the disagreement is possible.
  - There is a conflict of interest among players about agreements.
- Bargaining solution
  - An agreement or a disagreement



A simple example

- A simple example: Player 1 (seller) sells a book to Player 2 (buyer) at a price $p=?$.
  - Problem: Players bargain for the price p
  - The objective (payoff) of players: $u_1=p-0$, $u_2=1-p$
  - The set of feasible agreements: $U = \{(u_1,u_2) \mid u_1+u_2=1\}$
  - The disagreement: $D = (d_1,d_2)$, e.g., $D=(0,0)$
  - A bargaining solution is an outcome $(v_1,v_2) \in U \cup D$



What bargaining solution will emerge?

A simple example

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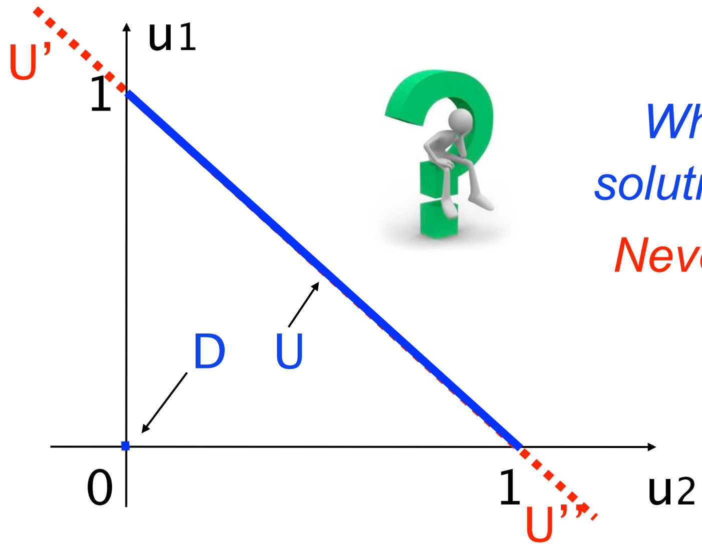

What bargaining solution will emerge?

Never in U' and U''.

A simple example

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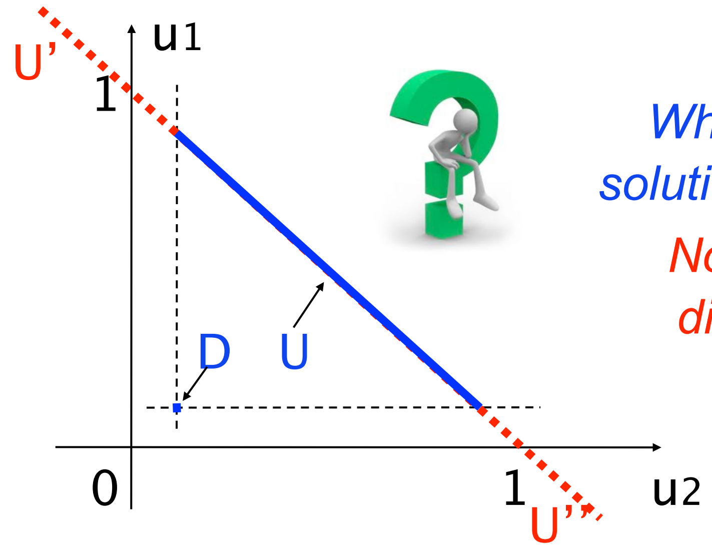
  - The objective (payoff) of players: $u_1=p-0$, $u_2=1-p$
  - The set of feasible agreements: $U = \{(u_1,u_2) \mid u_1+u_2=1\}$
  - The disagreement: $D=(d_1,d_2)$, e.g., $D=(0.1,0.1)$
  - A bargaining solution is an outcome $(v_1,v_2) \in U \cup D$



What bargaining solution will emerge?

No worse than disagreement

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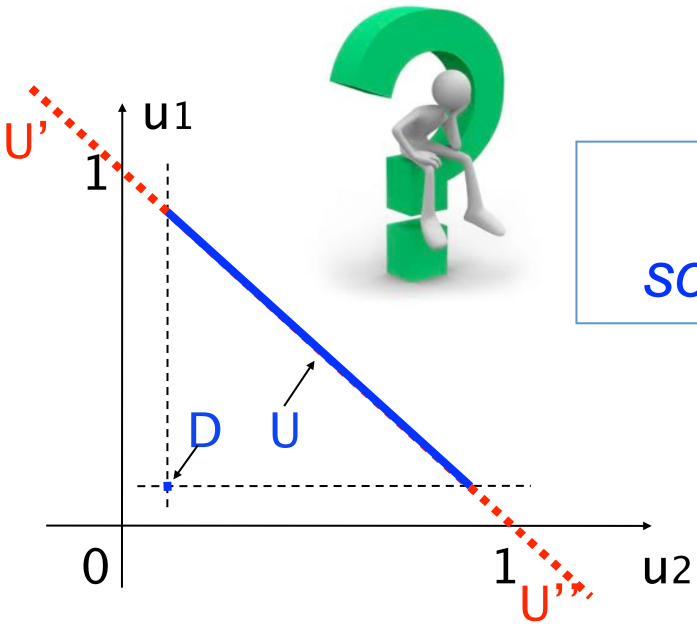
Bargaining Theory

- Bargaining theory is a theoretic tool used to identify the bargaining solution, given
  - (i) the set of all feasible agreements U
  - (ii) the disagreement D



What bargaining solution will emerge?

Bargaining Theory

- Axiomatic Approach
  - (i) Abstracting away the details of the bargaining process;
  - (ii) Considering only the set of outcomes that satisfy certain pre-defined properties (i.e., Axioms).
  - Typical Example: Nash Bargaining Model, 1950
- Strategic Approach
  - (i) Modeling the bargaining process as a game explicitly;
  - (ii) Considering the game outcome (i.e., Nash equilibrium) that results from the players' self-enforcing interactions.
  - Typical Example: Rubinstein Bargaining Model, 1982

Bargaining Theory

- Bargaining solution by axiomatic approach

Pre-Define Axioms

Axiom 1: Pareto efficiency
Axiom 2: Equal share of payoff gain
Axiom 3: Symmetry
...

↓

Bargaining Solution(s):

The solution(s) that satisfies all axioms

Bargaining solution is the solution satisfying all axioms.

Typical Example: Nash Bargaining Model, 1950
Shapley Bargaining Model, 1976
… …

Bargaining Theory

- Bargaining solution by strategic approach

Bargaining Game Formulation:

1-Stage Game
2-Stage Game
...
Infinite-Stage Game (Rubinstein)

↓

Bargaining Solution(s):

The Nash Equilibrium(s) of the game

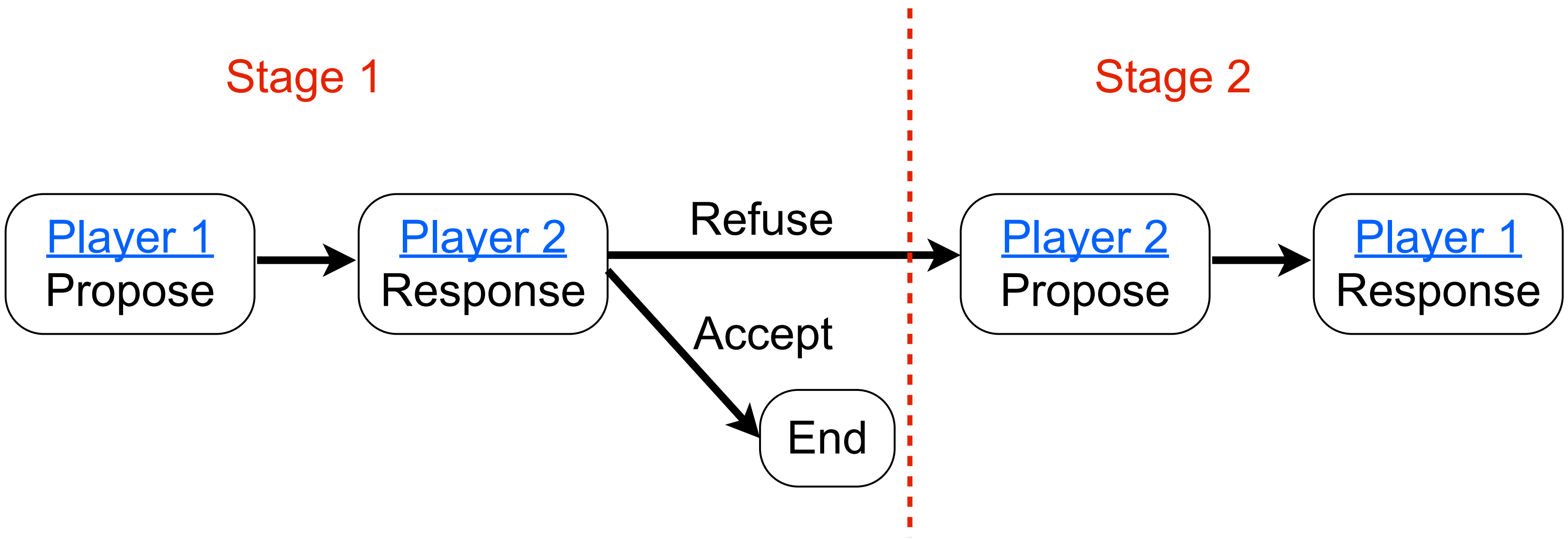
Bargaining solution is the Nash equilibrium of the game.
Typical Example: Rubinstein Bargaining Model, 1982

Bargaining Theory

- Bargaining solution by strategic approach
  - A possible 2-stage bargaining game formulation:
    - Stage 1: Player 1 proposes a solution (e.g., a price $p=p_1$ in the previous example), and Player 2 accepts or refuses;
      - If player 2 accepts, bargaining terminates at the proposed solution (agreement), otherwise, turn to Stage 2;
    - Stage 2: Player 2 proposes a solution, and player 1 accepts or refuses;
      - If player 1 accepts, bargaining terminates at the proposed solution (agreement), otherwise, bargaining terminates at the disagreement.

Bargaining Theory

- Bargaining solution by strategic approach



2-Stage Propose-Respond Bargaining Game Formulation

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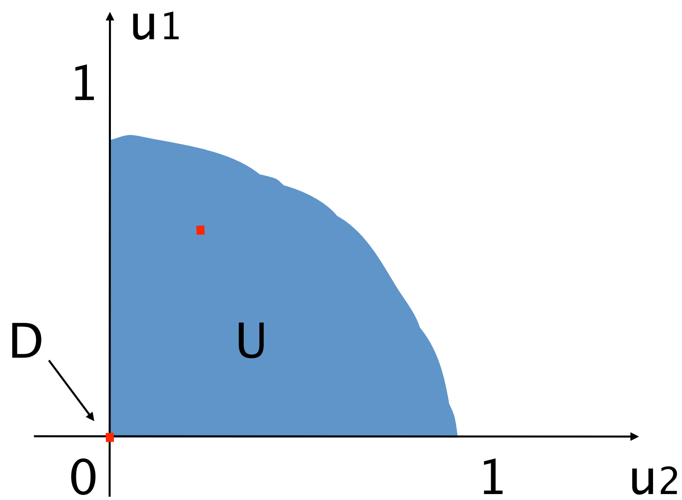
Bargaining Theory

- 2-person bargaining problem [Nash J., 1950]
- An axiomatic approach based bargaining solution
- 4 Axioms
  - (1) Pareto Efficiency
  - (2) Symmetry
  - (3) Invariant to Affine Transformations
  - (4) Independence of Irrelevant Alternatives
- Nash Bargaining Solution (NBS) is the unique solution that satisfies the above 4 axioms.

Nash Bargaining Model

- A general 2-person bargaining model
  - The set of bargaining players: $N = \{1,2\}$
  - The set of feasible agreements: $U = \{(u_1,u_2) \in$ a bounded convex set$\}$
  - The outcome of disagreement: $D = (d_1,d_2)$, e.g., D=(0,0)
  - A Nash Bargaining Solution is the unique outcome $(v_1,v_2) \in U \cup \{D\}$ that satisfies the Nash's 4 axioms.



Nash's Axioms

- Nash's 4 Axioms
  - (1) Pareto Efficiency: None of the players can be made better off without making at least one player worse off;
  - (2) Symmetry: If the players are indistinguishable, the solution should not discriminate between them;
  - (3) Invariant to Affine Transformations: An affine transformation of the payoff and disagreement point should not alter the outcome of the bargaining process;
  - (4) Independence of Irrelevant Alternatives: If the solution (v_1,v_2) chosen from a feasible set A is an element of a subset B $\subseteq$ A, then (v_1,v_2) must be chosen from B.

** Thought: Are these axioms reasonable?

Nash Bargaining Solution

- Nash Bargaining Solution (NBS) is the unique solution that satisfies the Nash's 4 axioms.

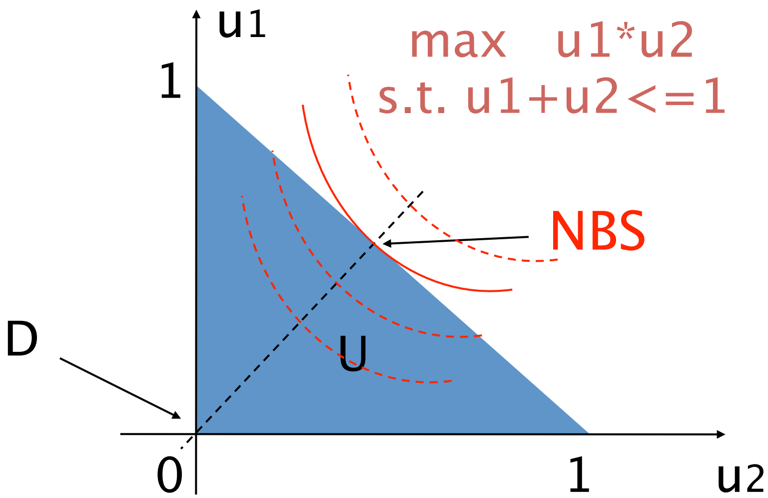
Definition

We say that a pair of payoffs (v_1^*, v_2^*) is a Nash bargaining solution if it solves the following optimization problem:

$$\max_{v_1, v_2} \quad (v_1 - d_1)(v_2 - d_2) \tag{1}$$
$$\text{subject to} \quad (v_1, v_2) \in U$$
$$(v_1, v_2) \geq (d_1, d_2)$$

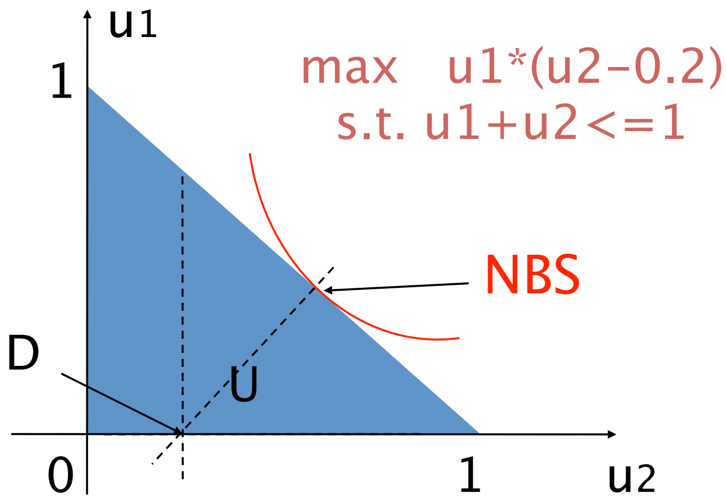
Nash Bargaining Solution

- An illustration of NBS: 2 players split 1 dollar
  - The set of feasible agreements: $U = \{(u_1,u_2) \mid u_1+u_2 \le 1,\ u_1, u_2 \ge 0\}$
  - The outcome of disagreement: $D = (d_1,d_2)$



(a) NBS when D=(0,0)

$(u1,u2) = (0.5, 0.5)$



(b) NBS when D=(0,0.2)

$(u1,u2) = (0.4, 0.6)$

Nash Bargaining Solution

- Important factors determining a NBS
  - Feasible agreement sets U
  - Disagreement D
    - Increase a player's disagreement ➔ higher payoff for the player in Nash Bargaining Solution.
  - Bargaining power **a**
    - Increase a player's bargaining power ➔ higher payoff for the player in Nash Bargaining Solution.

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Rubinstein Bargaining Solution

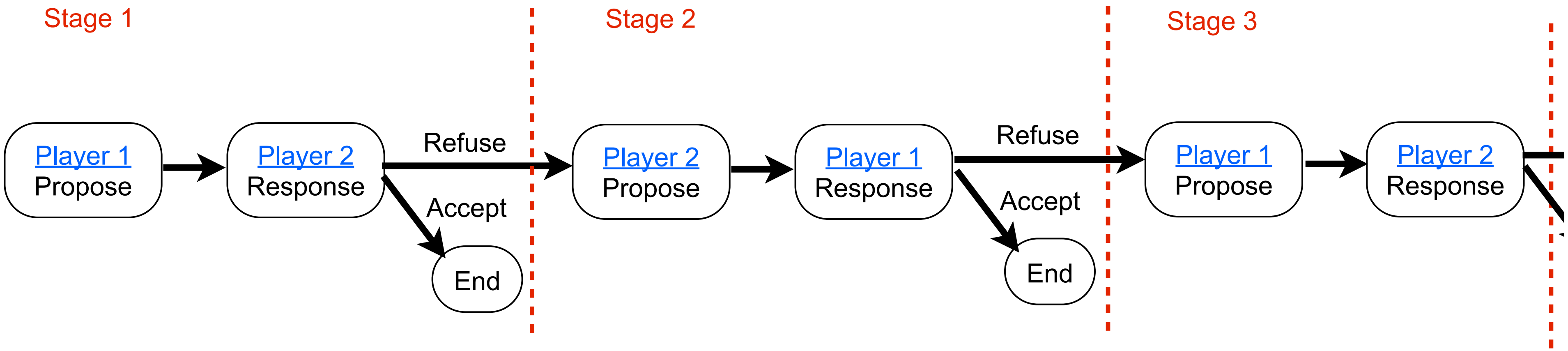
- 2-person bargaining problem [Rubinstein, 1982]
- A strategic approach based bargaining solution
- Bargaining Game Formulation

  -- Infinite-Stage Propose-Response Game
- Rubinstein Bargaining Solution (RBS) is the Nash equilibrium of the game.

Rubinstein Bargaining Solution

- Rubinstein Bargaining Game Formulation

  -- Infinite-Stage Propose-Response Game



Time Discount – The earlier an agreement is achieved, the higher the payoffs for both players.

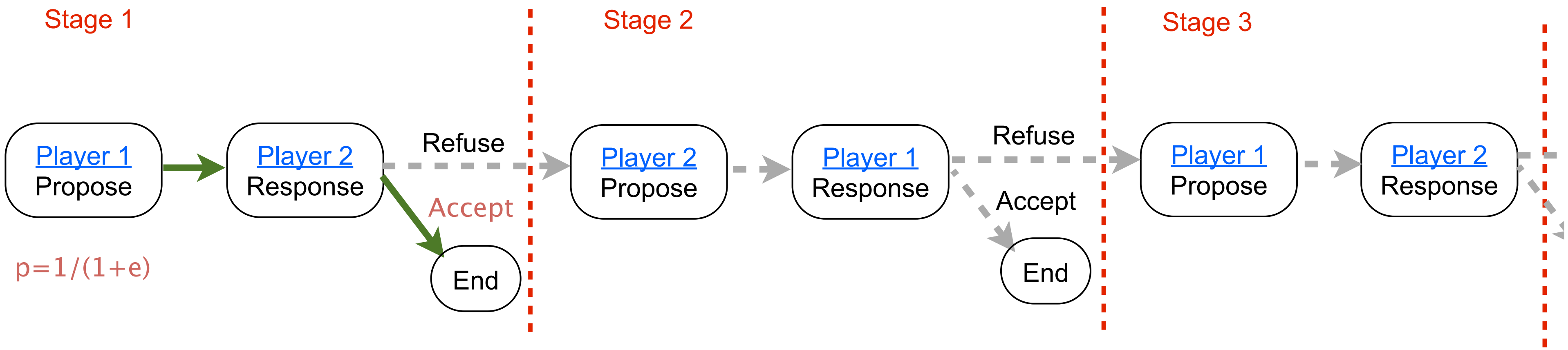
Rubinstein Bargaining Solution

- A simple example: player 1 (seller) wants to sell a book to player 2 (buyer) at a price p=?.
  - Problem: Players bargain for the price p
  - The objective (payoff) of players: $u_1=p-0$, $u_2=1-p$
  - The set of feasible agreements: $U = \{(u_1,u_2) \mid u_1+u_2=1\}$
  - The disagreement: $D = (0,0)$
  - A bargaining solution is an outcome $(v_1,v_2) \in U \cup D$
  - Time Discount
    - When achieving an agreement p at Stage t+1, the payoff of players are: $u_1=(p-0)*e^t$, $u_2=(1-p)*e^t$, where $0<e<1$.

Rubinstein Bargaining Solution

Nash Equilibrium of Rubinstein Game

- Player 1 proposes a price $p=1/(1+e)$ at Stage 1;
- Player 2 accepts immediately.
- Payoff of Players: $u_1=1/(1+e)$, $u_2=e/(1+e)$



** Question: How to derive this Nash Equilibrium?

Rubinstein Bargaining Solution

- RBS vs NBS

When e-->1, Rubinstein Bargaining Solution (RBS) is equivalent to Nash Bargaining Solution (NBS) !

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Conclusion

- We discuss the basic formulation of bargaining problem, and two classic approaches to the bargaining solution:
  - Axiomatic approach: Nash Bargaining Solution
  - Strategic approach: Rubinstein Bargaining Solution

Questions

- (p.18) Thought: Are these axioms reasonable?
  - Can you propose other possible axioms?
- (p.26) Question: How to derive this Nash Equilibrium?
  - Formulate the bargaining problem as a T-Stage (where T=1,2,...) Propose-Response game, and derive the Nash Equilibrium.

Thank you !