Nash Bargaining Solution and Application

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Networks: Technology, Economics, and Social Interactions

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Outline

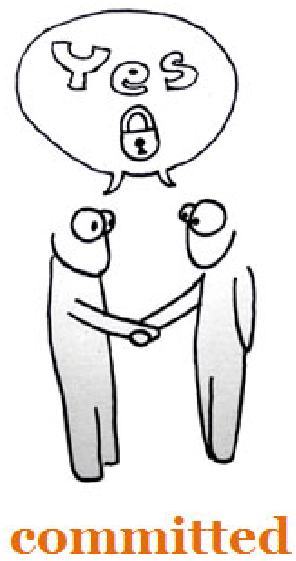
- Part I: Theory
  - Bargaining Problem
  - Nash Bargaining Solution
- Part II: Application
  - Case: Mobile data offloading
- Conclusion

Bargaining Problem

- Bargaining is one of the most common activities in daily life.
  - Price bargaining in an open market;
  - Wage and working time bargaining in a labor market;
  - Score bargaining after an examination;
  - … …

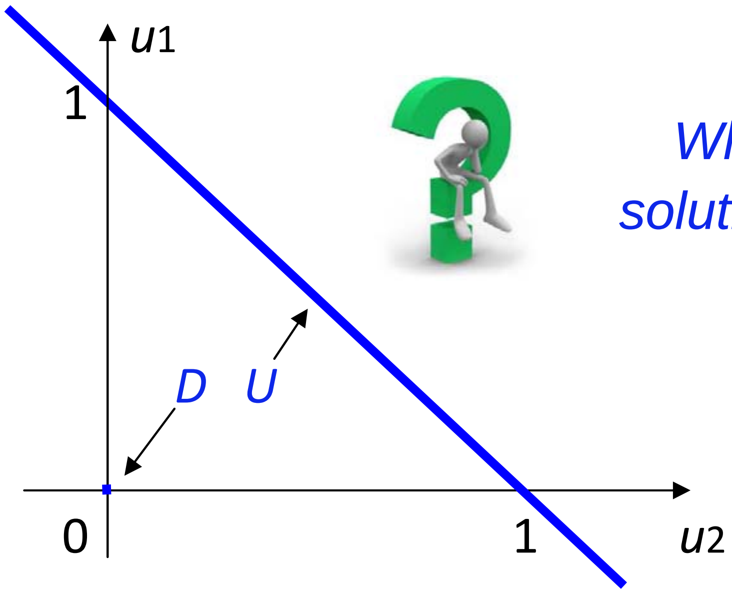
Bargaining Problem

- Formally, bargaining problems represent situations in which:
  - Multiple players with specific objectives search for a mutually agreed outcome (agreement).
  - No agreement may be imposed on any player without his approval, i.e., the disagreement is possible.
  - Players have the possibility of reaching a mutually beneficial agreement.
  - There is a conflict of interest among players about agreements.
- Bargaining solution
  - An agreement or a disagreement



A simple example

- A simple example: player *1* wants to sell a book to player *2*.
  - Problem: Players bargain for the price p
  - The objective (payoff) of players: $u_1 = p$, $u_2 = 1 - p$
  - The set of feasible agreements: $U = \{(u_1, u_2) \mid u_1 + u_2 = 1\}$
  - The disagreement: $D = (d_1, d_2)$, e.g., $D = (0,0)$
  - A bargaining solution is an outcome $(v_1, v_2) \in U \cup D$



What bargaining solution will emerge?

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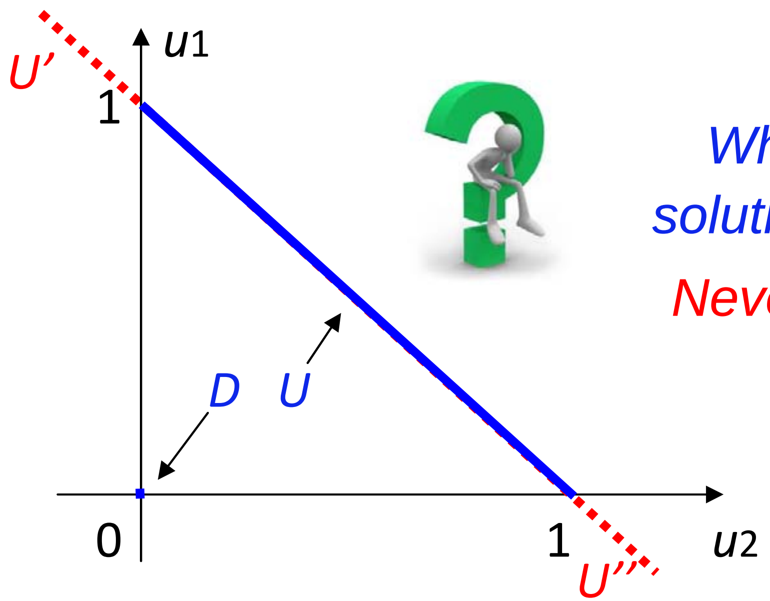

What bargaining solution will emerge?

Never in U' and U''.

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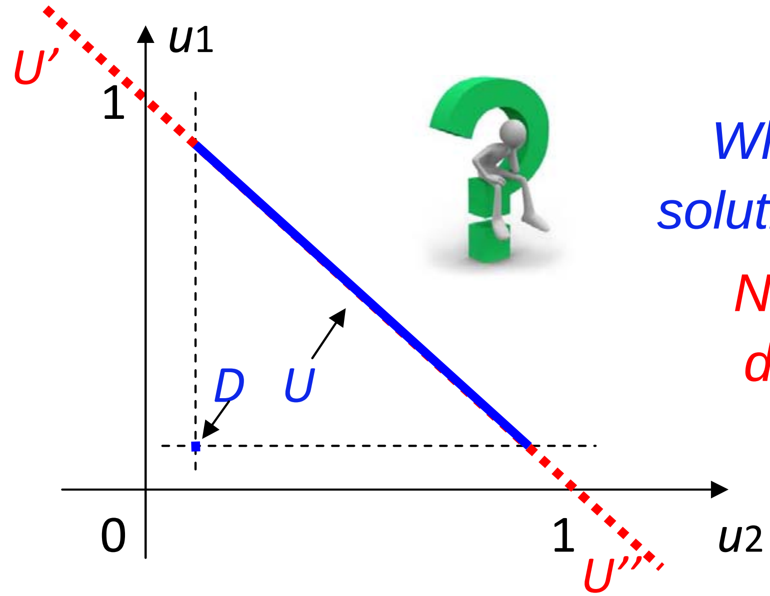
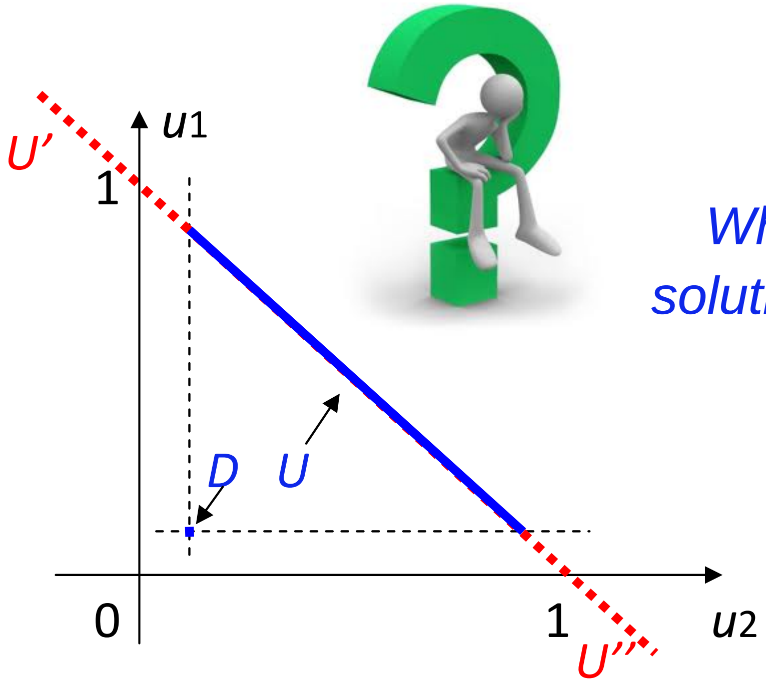


What bargaining solution will emerge?

No worse than disagreement

Bargaining Theory

- Bargaining theory is a theoretic tool used to identify the bargaining solution, given
  - (i) the set of all feasible agreements *U*
  - (ii) the disagreement *D*



What bargaining solution will emerge?

Bargaining Theory

- Strategic Approach vs Axiomatic Approach
  - Strategic approach: (i) Modeling the bargaining process (i.e., the game form) explicitly, and (ii) Considering the game outcome (i.e., equilibrium) that results from the players' self-enforcing interactions.
    - e.g., Rubinstein Bargaining Model, 1982
  - Axiomatic approach: (i) Abstracting away the details of the process of bargaining, and (ii) Considering only the set of outcomes or agreements that satisfy "reasonable" properties.
    - e.g., Nash Bargaining Model, 1950

Bargaining Theory

- Bargaining solution by strategic approach
  - A simple illustration: Player 1 wants to sell a book to player 2
  - Stage 1: Player 1 proposes a price $p=p_1$, and player 2 accepts or refuses;

    If accept, bargaining terminates; If not, turn to Stage 2;
  - Stage 2: Player 2 proposes a price $p=p_2$, and player 1 accepts or refuses;

    If accept, bargaining terminates; If not, turn to Stage 3;
  - Stage 3: Player 2 proposes a price $p=p_3$, and player 2 accepts or refuses;

    If accept, bargaining terminates; If not, turn to Stage 4;
  - … …

The bargaining solution is the equilibrium of this game.
Example: Rubinstein Bargaining Model, 1982

Bargaining Theory

- Bargaining solution by axiomatic approach
  - A simple illustration: Player 1 wants to sell a book to player 2
  - Axiom 1: Pareto efficiency.
  - Axiom 2: Equal share of payoff gain.
  - Axiom 3: …
  - … …

The bargaining solution is the solution satisfying all axioms.

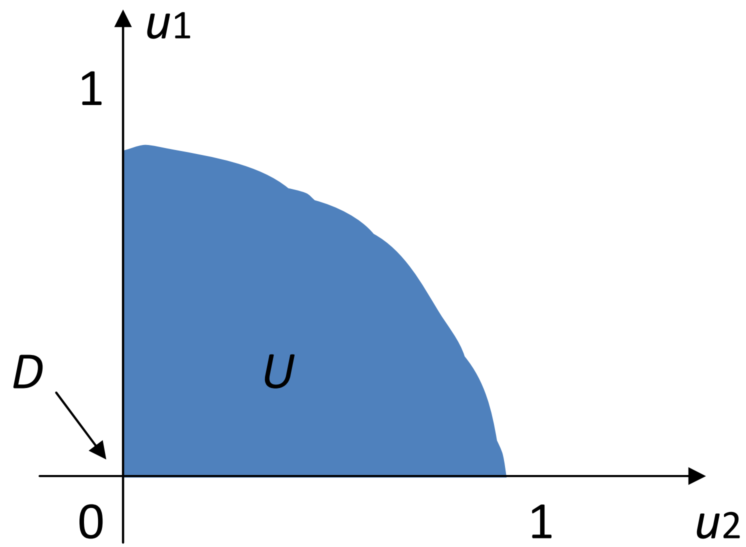
Example: Nash Bargaining Model, 1950
Shapley Bargaining Model, 1976
… …

Nash Bargaining Solution

- 2-person bargaining problem [Nash J., 1950]
- An axiomatic approach based bargaining solution
- 4 Axioms
  - (1) Pareto Efficiency
  - (2) Symmetry
  - (3) Invariant to Affine Transformations
  - (4) Independence of Irrelevant Alternatives
- Nash bargaining solution is the unique solution that satisfies the above 4 axioms.

2-person Bargaining Problem

- A general 2-person bargaining model
  - The set of bargaining players: $N = \{1,2\}$
  - The set of feasible agreements: $U = \{(u_1,u_2) \in \text{a bounded convex set}\}$
  - The outcome of disagreement: $D = (d_1,d_2)$, e.g., $D=(0,0)$
  - A Nash bargaining solution is an outcome $(v_1,v_2) \in U \cup \{D\}$ that satisfies the Nash's 4 axioms.



Nash's Axioms

- Nash's 4 Axioms
  - (1) Pareto Efficiency: None of the players can be made better off without making at least one player worse off;
  - (2) Symmetry: If the players are indistinguishable, the solution should not discriminate between them;
  - (3) Invariant to Affine Transformations: An affine transformation of the payoff and disagreement point should not alter the outcome of the bargaining process;
  - (4) Independence of Irrelevant Alternatives: If the solution (v_1, v_2) chosen from a feasible set A is an element of a subset $B \subseteq A$, then (v_1, v_2) must be chosen from B.

Thought: Are these axioms reasonable?

Nash Bargaining Solution

- Nash bargaining solution (NBS) is the unique solution that satisfies the Nash's 4 axioms.

Definition

We say that a pair of payoffs (v_1^*, v_2^*) *is a Nash bargaining solution if it solves the following optimization problem:*

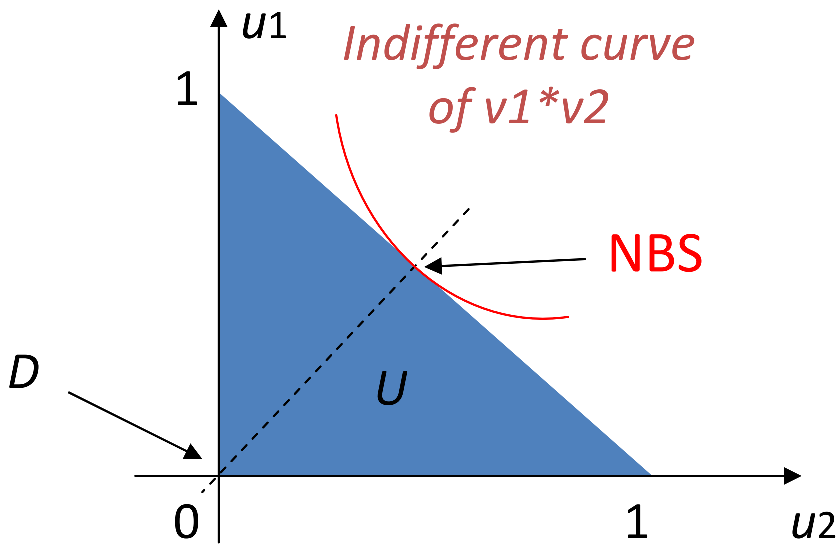
$$\max_{v_1, v_2} \quad (v_1 - d_1)(v_2 - d_2) \tag{1}$$

$$\text{subject to} \quad (v_1, v_2) \in U$$

$$(v_1, v_2) \geq (d_1, d_2)$$

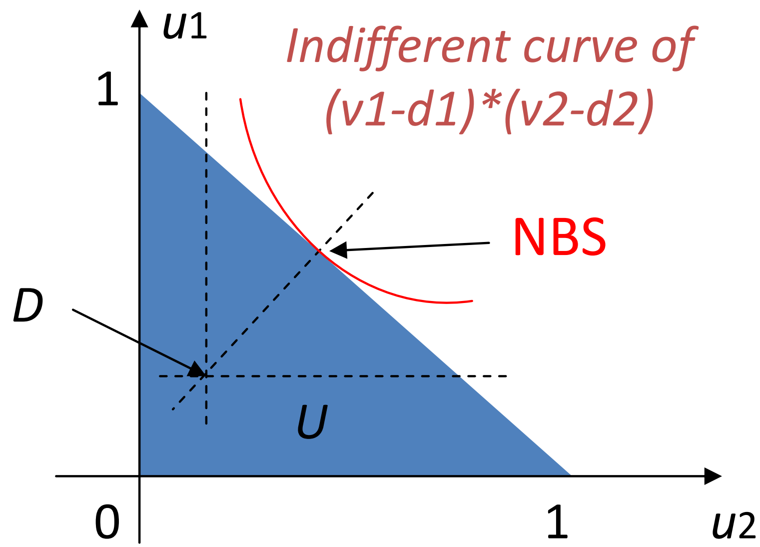
Nash Bargaining Solution

- An illustration of NBS: 2 players split 1 dollar
  - The set of feasible agreements: $U = \{(u_1,u_2) \mid u_1+u_2 <=1,\ u_1,\ u_2>=0\}$
  - The outcome of disagreement: $D = (d_1,d_2)$



(a) NBS when D=(0,0)

(v1,v2) = (0.5, 0.5)



(b) NBS when D=(0.3,0.2)

(v1,v2) = (0.55, 0.45)

Nash Bargaining Solution

- Important factors determining a NBS
  - Feasible agreement sets *U*
  - Disagreement *D*
    - Increase a player's disagreement → higher payoff for the player in Nash bargaining solution.
  - Bargaining power *a*
    - Increase a player's bargaining power → higher payoff for the player in Nash bargaining solution.

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Case: Mobile Data Offloading

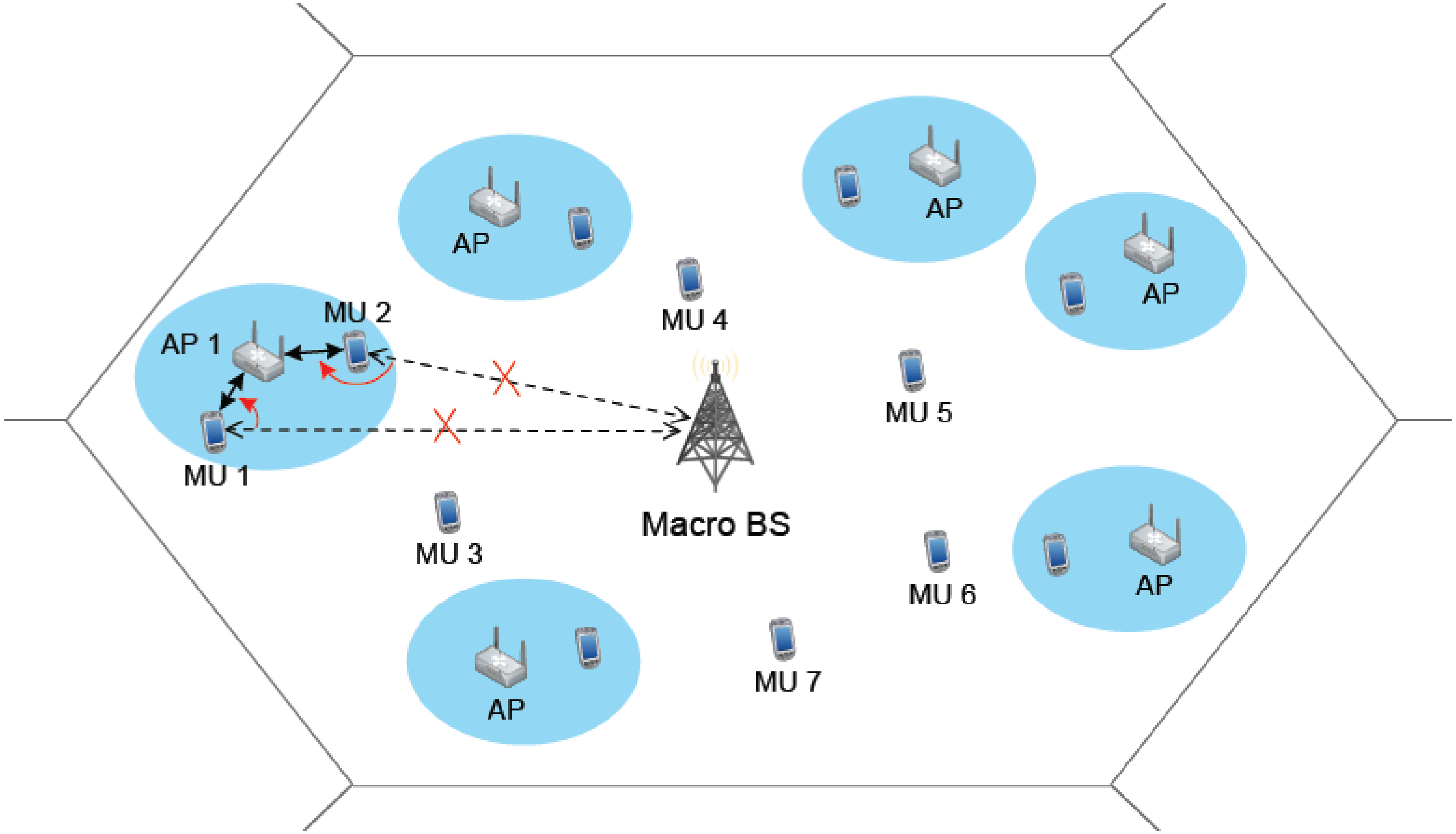


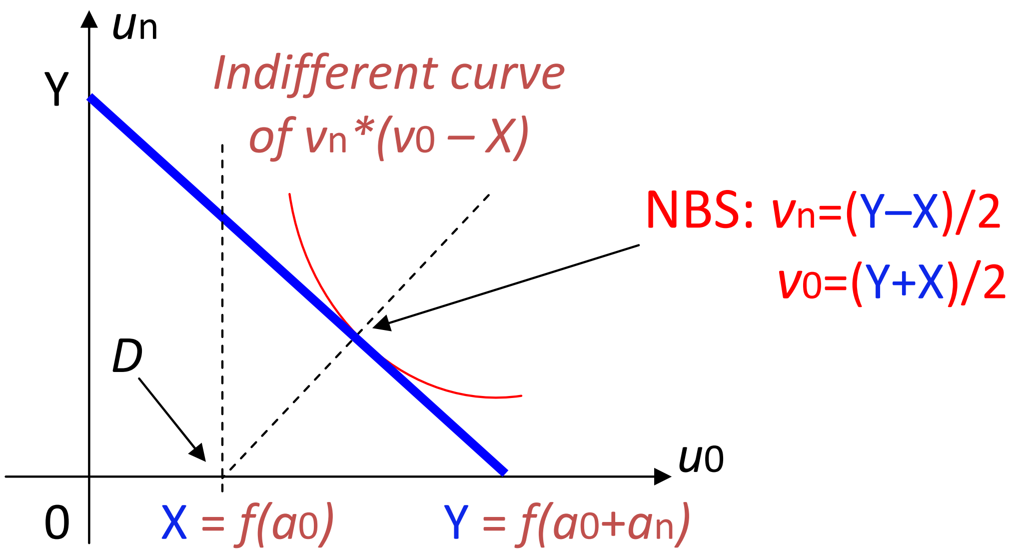
Figure. An illustration of mobile data offloading in a macrocell BS. A Mobile User (MU) can directly access the BS or offload his data (send or receive) through an AP if he is within both coverage areas.

Bargaining in Data Offloading

- BS wants to buy spectrum resource from APs
  - Problem: BS and each AP *n* bargain for the price p_n
- Key notations
  - The amount of AP *n's* resource: a_{n}, $n=1,2,\ldots,N$
  - The price for AP *n's* resource: p_{n}, $n=1,2,\ldots,N$
  - The amount of BS's own resource: a_0
  - The welfare function of BS: $f(a)$ (strict increasing)

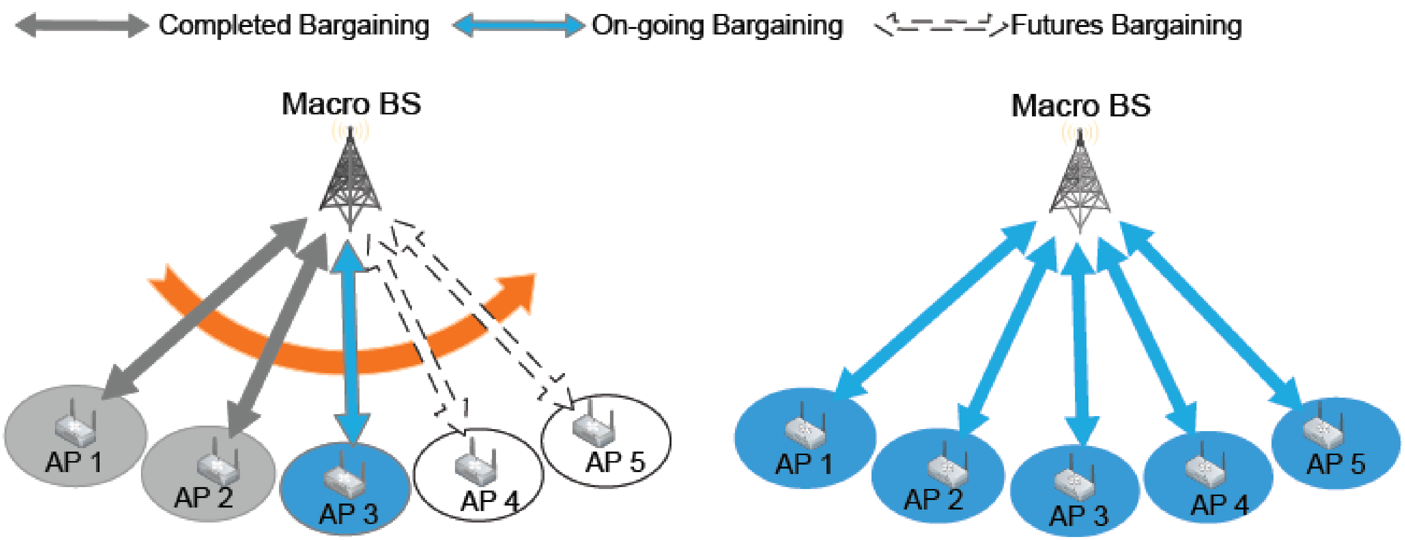
Bargaining in Data Offloading

- An illustration of bargaining between AP *n* and BS
  - 2-person bargaining problem
  - The objective (payoff) of players: $u_n = p_n$, $u_0 = f(a_0 + a_n) - p_n$
  - The set of feasible agreements: $U = \{(u_n, u_0) \mid u_n + u_0 = f(a_0 + a_n)\}$ (blue curve)
  - The disagreement: $D = (0, f(a_0))$



Bargaining in Data Offloading

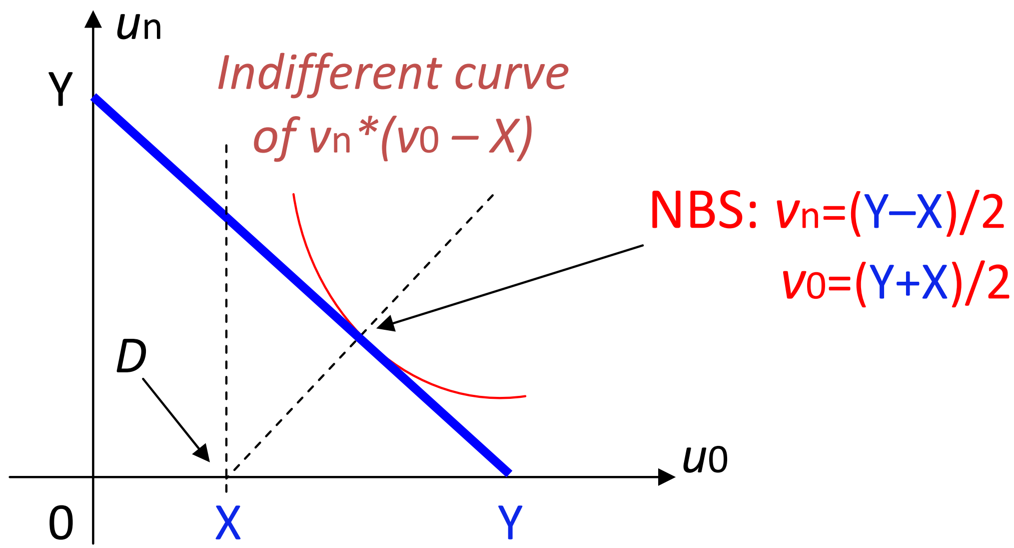
- Totally *N* two-person bargaining problems
- Bargaining protocol:
  - Sequentially bargaining
  - Concurrently bargaining



(a) Sequential Bargaining (b) Concurrent Bargaining

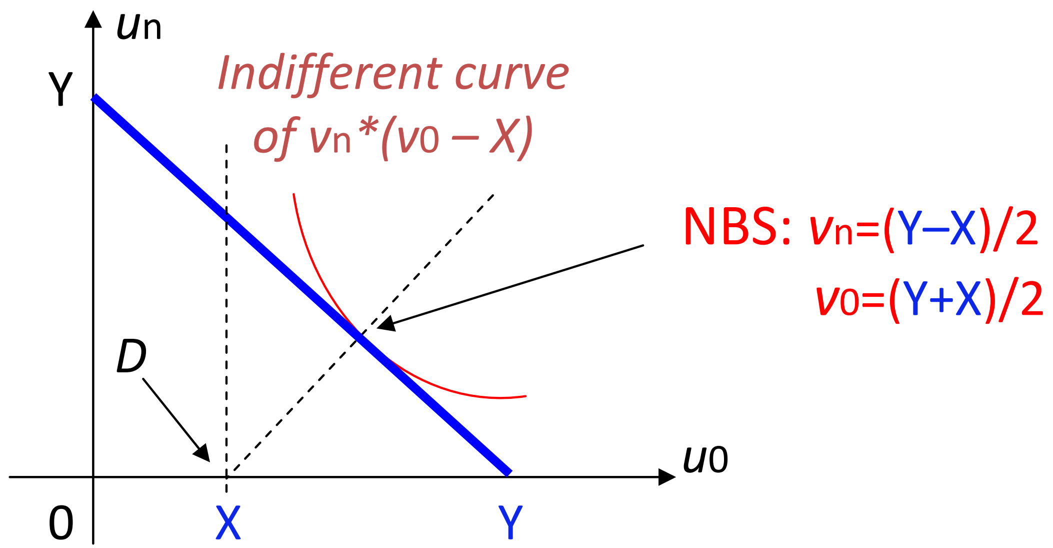
Bargaining in Data Offloading

- Sequentially Bargaining in step *n*: BS and AP *n*
  - 2-person bargaining problem
  - The objective (payoff) of players: $u_n = p_n$, $u_0 = f(a_0+a_1+\ldots+a_n) - p_n$
  - The set of feasible agreements: $U = \{(u_n, u_0) \mid u_n + u_0 = f(a_0+a_1+\ldots+a_n)\}$
  - The disagreement: $D = (0, f(a_0+a_1+\ldots+a_{n-1}))$



Bargaining in Data Offloading

- Concurrently Bargaining between BS and AP *n*
  - 2-person bargaining problem
  - The objective (payoff) of players: $u_n = p_n$, $u_0 = f(a_0 + a_1 + \ldots + a_N) - p_n$
  - The set of feasible agreements: $U = \{(u_n, u_0) \mid u_n + u_0 = f(a_0 + a_1 + \ldots + a_N)\}$
  - The disagreement: $D = (0, f(a_0 + a_1 + \ldots + a_{n-1} + a_{n+1} + \ldots + a_N))$



Bargaining in Data Offloading

- Examples: 10 APs
  - The amount of AP *n's* resource: $a_n = 1$, n=1,2,…,10
  - The amount of BS's own resource: $a_0 = 5$
  - The welfare function of BS: $f(a) = \log(a)$
- NBS in Sequentially Bargaining
  - APs: $u_1 = 0.5*\log(6) - 0.5*\log(5)$, $u_2 = 0.5*\log(7) - 0.5*\log(6)$, …,

    $u_{10} = 0.5*\log(15) - 0.5*\log(14)$,
  - BS: $u_0 = 0.5*\log(15) + 0.5*\log(5)$
- NBS in Concurrently Bargaining
  - APs: $u_1 = 0.5*\log(15) - 0.5*\log(14)$, $u_2 = 0.5*\log(15) - 0.5*\log(14)$, …,

    $u_{10} = 0.5*\log(15) - 0.5*\log(14)$,
  - BS: $u_0 = 5*\log(14) - 4*\log(15)$

Conclusion

- We discuss the basic theory of bargaining solution, in particular the Nash bargaining solution.
- We discuss a potential application of Nash bargaining solution in wireless networks: mobile data offloading.

Thank you !